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ABSTRACT

This chapter presents a yearlong teaching experiment involving a 4th grade class in a New Jersey school that focused on fractions. The 25 children worked in pairs or in small groups, then came together as a whole class for sharing in larger discussions. The development of children's thinking is explored through analyzing videotapes of class sessions, studying the children's written work and researcher's field notes, and listening to students' conversations as they discussed their ideas with one another. Results indicate that students built important understanding of fractions by modeling, drawing pictures, and eventually developing the notation for their ideas. (KHR)

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Young Children's Growing Understanding of Fraction Ideas

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CAN young children learn to understand fractions? This was a question that led us to conduct a yearlong teaching experiment with a fourth-grade class in a New Jersey school. The twenty-five children were heterogeneously grouped, representing a wide range of abilities. They worked on the problems two or three times a week, beginning in late September. The classes were 60 to 80 minutes in duration. We shall report on the first seven of approximately 25 classroom sessions, from September through December, that focused on investigations involving fractions. For all these seven sessions, the children had available Cuisenaire Rods™ to build models. The children worked in pairs (or in small groups) and then came together as a whole class for sharing in larger discussions.

Our team's goal was to pose tasks to the children and to study how their thinking developed by analyzing the videotapes of each session, by studying the children's written work and researchers' field notes, and by listening to students' conversations as they discussed their ideas with one another. Decisions about follow-up activities were based on our best estimates of the children's progress. We also invited children to revisit the same or equivalent problems days or even weeks later. Because we wanted the children to be responsible for determining the reasonableness of their ideas, we encouraged them to decide on the correctness of their solutions and to think about making appropriate generalizations. Our expectations were that each child, or group of children,

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would construct models of their solutions, and that they would compare these models and their ideas with others. We encouraged them to move about the room and talk to one another. We asked them to prepare their solutions for class sharing and provided overhead rods, transparencies, and overhead projector pens in assorted colors. We worked to promote a classroom culture that supported individual initiative, explanation, and reflection.

In the description that follows, T1 and T2 represent the teacher-researchers. We will use symbols to refer to color and length of each rod: W – white (1 cm), R – red (2 cm), LG – light green (3 cm), P – purple (4 cm), Y – yellow (5 cm), DG – dark green (6 cm), BK – black (7 cm), BR – brown (8 cm), B – blue (9 cm), O – orange (10 cm) (See fig. 6.1). More rod activities are presented in Steencken (1999).

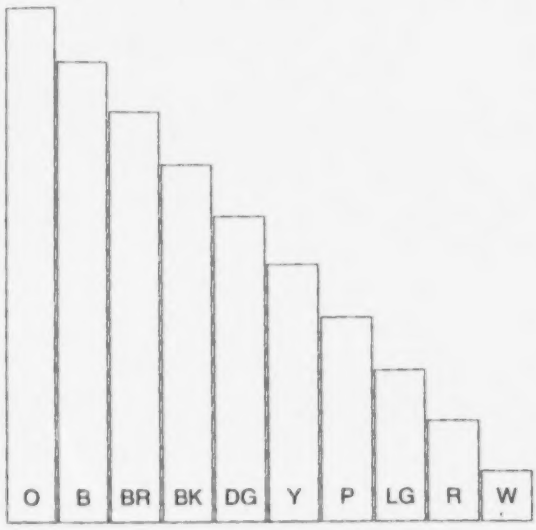

Fig. 6.1. Cuisinaire Rods™

SESSION 1

The first session consisted of a combination of free play with the rods and activities designed to help students distinguish between permanent color names for rods and variable number names. Students were invited to develop ways of supporting their solutions.

Some example activities: "I claim the light green rod is half as long as the dark green rod. What do you think? What would you do to convince me? What number name would we give to the light green rod if I called the dark green rod one? Someone told me that the red rod is half as long as the yellow rod. What do you think?" Students were also invited to pose problems to one another. For example, Alberto challenged, "If the red rod is considered one fifth, what would the orange rod be?" Betty and Mario asked, "If light green is one whole, what is blue? If blue is one, what is light green?

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SESSION 2

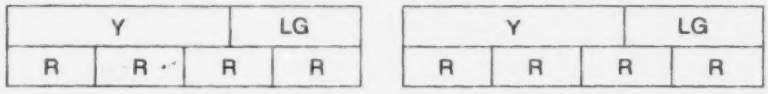
T1 began by asking the children what occurred in the previous session. Ed began: "Well, let's see. [He chose a blue rod.] If we said that the blue rod would be one whole, we'd figure out what ... we'd take all the blocks and try and figure out what would be one half of it."

The children were invited to find a solution. They concluded that no solution existed for finding a rod that is half as long as the blue rod within the set of rods. Matt and his group explained that for any set of rods, another rod, one half as long, would not exist, that is, the smallest rod in the set.

SESSION 3

The notion of making a "train" of rods—that is, making new rods by placing existing rods end to end—was introduced. Two questions were posed: "If a train of a yellow and a light green rod has the number name *two*, what number name will we give the red rod?"; and "If a train of a yellow and a light green rod has the number name *one*, what number name will we give the red rod?"

Children placed four red rods under their train of yellow and light green rods. Siham and Aleta presented their models using overhead rods and built two identical models with different number names (see fig. 6.2).



Train of yellow and light green is 2.
What is the number name for red?

Train of yellow and light green is 1.
What is the number name for red?

Fig. 6.2. Students' solution models

The school principal, who frequently visited classes and interacted with the children, asked Aleta to explain the model she built with Siham.

> *Aleta:* Oh, first we put the red rods up to the yellow and light green rods, and then we said, "If the yellow and the green was two, what would we call the red rod?" And we thought that we would call it one and one fourth. And if it [the train] was one, we would call it one fourth.

Principal: Okay, so if it was one, you said you would call it one fourth, and if it was two, what did you say?

Aleta: It would be one and one fourth.

T1 asked the class if they agreed with Aleta's solution. The children indicated that they did not. T1 then asked the girls what they would have to do to convince the class, requesting that the girls first explain the train whose length had the number name *one*. In this train, the girls explained that they named the red rod *one fourth* because there were four red rods whose length together equaled to the length of the train. The class agreed with the girls' name for the red rod when the train was given the number name *one;* but they disagreed with the name for the red rod when the train was given the number name *two*. Bob and Janet pointed out that the red rod should be called *one half*. Dan explained why the red rod should be given the number name *one half:*

> Okay, so this is two [the train given the name *two*], and this would be a half [a red rod] because if you put another one and another one that'd be two. [He aligned four red rods on the overhead.] And if you take away these [two red rods] that would be one and took away that [another red rod], leaving one red rod, that would be a half.

T1 asked the class what they found so confusing about this problem. Ed offered the following explanation:

> I think the confusion is, they think ... that they think ... they have the temptation of calling ... since there are four red blocks, they think they are gonna call it one fourth 'cause they forgot that the yellow and the [light] green are two.... Because, see, if you have one, there'd be two halves; but if you have two it's two halves plus two halves, which would be four halves. Therefore, you would have to call one of the reds one half.

T1 then told the class that she had given to each of two people, Tom and Amy, half of a chocolate bar. Tom was happy with his piece but Amy complained that her amount was unfair. T1 asked how that could have happened, and Mario replied: "You probably gave Tom a bigger half than Amy." T1 asked if that made sense. Meredith responded with an example using yellow and light green rods. She said: "Well, see this was *one* [the yellow and light green train], and then you gave this much to Tom [pointing to the yellow rod, which was five eighths] and this much to Amy [pointing to the light green rod, which was three eighths], that wouldn't be a fair cut."

Dan said that he agreed with Meredith and pointed out that a half should be "even." T1 continued with the candy-bar metaphor. She displayed a large and a small candy bar and said that she gave half of the large one to Tom and half of the small one to Amy. The children giggled and conceded that it was unfair to talk about halves with different wholes.

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Next, a comparison problem involving unit fractions was posed to the class: "Which is larger, one half or one third?" T1 invited the students to comment on the meaning of the problem. Matt responded, "Well, normally, one half is bigger than one third, but if you got a bigger size of candy bar or pizza, and if you get one third of that, then that'd be more than one half of a little pizza."

T1 held up a candy bar [scored in a three by four grid]. She asked the children to consider which was bigger, 1/2 or 1/3, in relation to the candy bar. Children worked on a solution and built various models with their rods. The class ended with children agreeing that 1/2 was bigger than 1/3. When all were asked to give a number name for the difference, Alberto shared a model (see fig. 6.3) in which *one* was represented by a train of an orange and red rod. He said, "We know already, that, that, three reds would make a dark green and if there are two dark greens to make an orange and the red rod, then it would take six red rods to make the orange and the red rod."

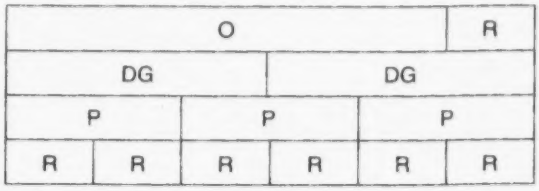

Fig. 6.3. Alberto's model

Various models were built by the children who were convinced that 1/2 was bigger than 1/3; however, they were still working to find a number name for the difference.

SESSION 4

Children were asked to investigate specific relationships between the rods and assign appropriate number names to the rods. T2 asked, "If I call the orange rod one, what number name will I give to the white rod?" The children named the white rod *one tenth*. T2 continued, "If I call the orange rod one, what number name will I give to the red rod?" The children named the red rod *one fifth*. T2 then asked, "If I call the orange rod one, what number name will I give to two white rods?" Mario and Art answered, "One fifth." They went to the overhead to present their solution, which showed the length of the two white rods as equal to the length of the red rod. They gave the red rod the number name *one fifth*.

T2 then asked if there were other answers. In response, Meredith, Betty, Siham, and Dan presented their model, which named the orange rod *one*. The students placed 10 white rods under the orange rod and Meredith and Betty explained:

Meredith: I think it's two tenths. Take the red away and put this [white rod] up to the orange. When we did it before, we said that orange measures ten whites. If you put the whites up it would have ten. Two of ten is two tenths.

Betty: Since ten of these [white rods] equal one orange, then if you took two of these it would be two tenths because one equals one tenth and you just count one more and then you have two tenths.

T2 asked the class how there could be 2 answers, 2/10 and 1/5. Bob showed 2 white rods in one hand and one red rod in the other and explained:

Bob: Even if two white cubes equal up to one red cube, it's still not like imagining that this was another red cube, so I think it's two tenths because it actually is two tenths.

T2: Because you can see two there?

Bob: Yeah, I also see one fifth, but what you're seeing right here is two tenths, not a fifth.

T2 asked the class if it were possible for the two white rods to have both number names. Meredith repeated that the two white rods should be called *two tenths,* and said: "There's only two of them. They're not joined together. If you wanted to join them together you should use a red."

The children were dealing with the issue of equal lengths being represented by physical quantities with different names. Bob and Meredith agreed that the length that Art and Mario called *one fifth* was the same as the length that they called *two tenths.* The distinction was in the number of pieces, "two" for tenths, using white rods; "one" for fifths, using the red rod.

For the remaining 10 minutes of the session, the children were asked to think about the problem from the last session: "Which is bigger, one half or one third, and by how much?" They again agreed that one half was larger than one third. Except for Alberto, they had not spoken of a number name for the difference.

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SESSION 5

T1 presented on the overhead the following problem: "Does 1/5 = 2/10?" Meredith came to the overhead and built a model using an orange rod to represent one, and placed two yellow rods beneath the orange rod and explained that the yellow rod would have number name *one half.* She then placed five red rods underneath two yellow rods. She put two white rods together on the screen and explained that two white rods would be named *tenths,* whereas the red rod would be named *one fifth,* indicating that the same lengths can be measured with different units, tenths and fifths.

Meredith: And if you take one of them [She moved one red rod above the two white rods] it is equal to two tenths.

T1: So what is your conclusion if I ask you the question, is two tenths equal to one fifth?

Meredith: Yes.

Ed and Bob repeated Meredith's explanation to the rest of the class.

T1 produced an overhead transparency of a candy bar [scored in a 3 by 4 grid] and asked the children to give number names for 1/2 of the bar. Janet answered, "... six twelfths, because there are twelve pieces in all, and she got six, and six makes a half." Donna offered, "Two fourths ... because if she got a half, then the top two rows, um, is a half, and then that's two fourths." Bob added, "... three sixths. Yeah, because there are six of them—there, there, there. I found groups of sixths."

T1 then posed the earlier problem: "Which is bigger, one half or one third, and by how much?" Jenny and Lisa volunteered to give their solution at the overhead. They used a train of orange and red rods for *one.* They explained that 1/2 was bigger than 1/3. The teacher asked by how much, and the girls responded that it was a *red* bigger. When asked to give the red rod a number name, Aleta, who joined the other two girls, responded, "It's a third bigger, I think." Jenny and Lisa agreed, and Jenny explained, "I think it's one third bigger, too, because if you put the red to the [dark] green, you need three, and if you put the purple one to it also, and then it takes one third of them." Jenny measured the length of the red rod in comparison to the length of the dark green rod, which was one half in her model. Kate agreed. Bob did not, and said, "Well, I don't really agree. Well, if you split one of the thirds in half, which would make ... which would make a sixth. I think it's a sixth bigger."

T1 asked the class to think about what ideas were being presented. Janet offered a model in which the dark green rod was named *one,* the light green rod was named *one half* and the red rod was named *one third.* She stated, "the light green [*one half*] is one white bigger. After some discussion, the

girls agreed that the name of the white rod was *one sixth*. T1 then returned to the original model, where the train of orange and red rods was named *one*. Jenny held to her answer that the difference [a red rod] would be called *one third*. Bob went to the overhead and disagreed with Jenny, who responded, "Well, I think they might both be answers [*one sixth* and *one third*]." When asked, some children agreed with Jenny, some did not. Ed questioned, "How can you have one half be bigger than the thirds by one third? It has to be one sixth." Class ended as T1 asked the children to write about the different arguments that were presented.

SESSION 6

The session began with further discussion of the magnitude of the difference between 1/2 and 1/3. Some children expressed that they could provide a convincing argument that the difference must be 1/6. Jenny offered a model representing *one*, using a train of an orange and a red rod and showed the difference to be 1/6. Alberto's model made use of a dark green rod to represent *one* and also showed the difference to be 1/6. Jenny challenged Alberto's model, "Like remember, you [T1] said that it can be only one size candy bar, and that's it." Janet interjected, saying, "There can be candy bars of different sizes;" Alberto added, "You just can't switch the candy bar."

T1 asked, "What rod has the number name *one sixth* if we call the orange and red [train] *one*?" Siham answered, "Red." T1 then asked the children to name the white rod in this model. Various answers were suggested: *one sixth* [Juan], *one twelfth* [Bob], and *one tenth* [Lisa]. After a lively discussion, Juan changed his mind and went to the overhead. He placed 12 white rods under the train and explained that each white rod would be called 1/12. The children concurred.

A second problem was posed: "Which is bigger, one half or one fourth, and by how much?" The children built a variety of models. Janet, Alice, and Juan went to the overhead and built a model that used a brown rod as *one*, and two purple rods, each as a half. They placed eight white rods under the purple rods and called a white rod *eighths*. They measured the difference between the purple rod [1/2] and two white rods [1/4] as one white rod. They stated that 1/2 was bigger than 1/4 by 1/8. Meredith came to the overhead and showed the group that they still had room in their model for one more white rod, which would make the difference 2/8, not 1/8. She explained that the difference was 2/8 or 1/4, saying, "Yes. Okay. This is an eighth [one white rod]. It's not one eighth 'cause there's still a space. We're calling that an eighth [the white rod]. If you take another one it, um, it could be bigger by two eighths and, or it could be bigger by one quarter. One quar-

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ter or two eighths. It's the only way it could be one half." The children concurred.

SESSION 7

The problem, "Which is bigger, two thirds or one half, and by how much?", was presented. Some children immediately produced a model to show their solution; they were then asked if they could build more than one model. Meredith, who built her models in the order shown in figure 6.4, explained her solution to T2.

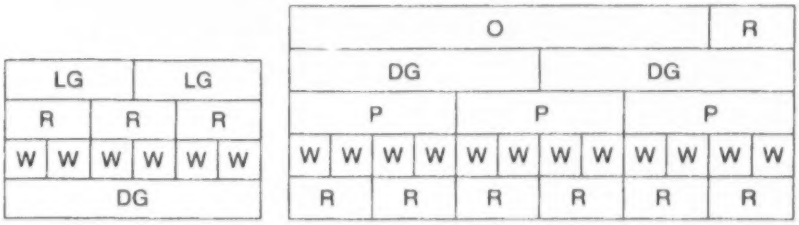

Fig. 6.4. Meredith's models

Meredith chose two white rods to measure the length between 2/3 and 1/2 in Model 2.

> And then you take the two thirds [she removed two purple rods and one dark green rod from Model 2], and you take two twelfths [the white rods] and then you put it up to two thirds and it is bigger than it by two tens... two twelfths.

Meredith then explained the difference, using Model 1.

> You call these [white rods] sixths, and you put this one [white rod] up to it [one light green rod she removed from Model 1], and it is bigger than [the two red rods she removed from Model 1] one sixth.

T2 asked if there was another number name for the difference between 2/3 and 1/2 in Model 2. Meredith responded.

> Um, yeah, well, maybe ... [Meredith placed six red rods below the train of 12 white rods, and placed a red rod above two of the train of 12 white rods.] ... Yeah, and maybe since two of these little white ones equals up to one of these. [She put one red rod on top of two white rods in the train, showing that a red rod is the same length as a train of two white rods.] Or it's one fifth. Oh, I mean one sixth.

Shortly after this exchange, T1 asked the class to share their ideas with one another. Janet, Ellen, and Jenny presented their solution to the entire class at the overhead and displayed the model that they built. Their model used the same rods for representing *one* as Meredith's Model 2 (see fig. 6.4). They used two dark green rods to show halves, three purple rods to show thirds, and six red rods to show sixths. One red rod was used to show the difference between 1/2 and 2/3. They named the red rod *one sixth*. The teacher researcher asked if there were any other solutions. Meredith responded eagerly and joined the girls. She placed 12 white rods under the six red rods displayed on the overhead. Meredith chose two white rods to measure the difference. She stated that 2/3 was larger than 1/2 by 2/12. Matt strongly objected, "No. ... No, they can't do that. Because the, the two thirds are bigger than a half by a red. So they can't use those whites to show it." T1 questioned Matt's modeling of the problem and asked him for his solution.

T1: And you got two thirds to be bigger than one half?

Matt: [Politely impatient] Yes.

T1: By how much?

Matt: [Deliberately] By one sixth.

Meredith: Or, or two twelfths.

Matt: [Shaking his head in dissent] No. [Mutterings in the classroom of "no"]

Meredith defended her idea and provoked a lively debate among her classmates. Ed offered his understanding of Meredith's solution.

Ed: Yeah, but see just the two whites together. That's right, it would be two twelfths. But you have to combine them. You can't call them, you can call them separately, but you can also call them combined and if you combine them it would be a, a, one sixth.

T1 questioned further.

SS: There's two answers.

Matt: [Simultaneously with Ed] No, they're the same answer.

Ed: No, they're the exact same thing, except she, she took the red and divided it into half, she divided it into halves, into half and called, and called each half one twelfth. They're the exact same answer except they're just in two parts.

T1: So we're talking about the length of the red rod: the length of the red rod is the same as the length of the two white rods? Is that true? Do you all agree to that?

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SS: Yeah. Yes.

Ed: And she's calling a white rod one twelfth and the other white rod one twelfth, and the red is really one sixth. Well, when she calls them two twelfths, the two twelfths are actually just two white rods put together to equal a red, so it should be really, it's really one sixth...

Ed offered his understanding of the debate.

Yeah. But I don't really think you could call, call them two twelfths because two twelfths equal exactly to the same size as one sixth. Well, if you want to you could call them, I guess. But I think it would be easier just to call them one sixth cause you wouldn't want to exactly call them one twelfth and another twelfth. I'd just call them one sixth. Therefore I think you just really call them one sixth.

CONCLUSIONS

How did the children's understanding of fractions grow? In the seven sessions, children built some very important understandings about fractions. They built models, drew pictures, and eventually developed notation for their ideas. They began with part-whole relationships and explored patterns created by successive "halving" first with a finite set of rods and later with the idea of infinitely many rods. They focused on the importance of naming the unit in their investigations. In comparing fractions, children developed a firm understanding that units used to represent fractions must be the same size. From rods they moved to other objects such as candy bars and pizzas. Matt commented, "Well, normally, one half is bigger than one third, but if you got a bigger size of candy bar or pizza, and if you get one third of that, then that'd be more than one half of a little pizza." The importance of using the same unit in comparing and finding the difference between fractions was the focal point in their investigations. Alberto reminded us that when comparing fractions you cannot switch the units: "You just can't switch the candy bar."

Children determined the meaning of equivalent number names as they discussed questions such as, "Is two tenths really the same as one fifth?" They explored more possibilities for comparing and naming differences as they built multiple models for their solutions. As the children moved to more-complex problems they made reference often to the rods, even thought they were not building models with them. In discussing their solutions, children listened to one another and developed convincing arguments to support alternative ideas. They eagerly and enthusiastically shared their

thinking and provided justifications about the validity of their solutions. They often raised questions and challenges that triggered further exploration and lively conversations. It is clear that young children can build a deep understanding of fraction ideas and enjoy doing it.

REFERENCES

Davis, Robert B. *Discovery in Mathematics: A Text for Teachers.* New Rochelle, N.Y.: Cuisenaire Company of America, 1980.

Maher, Carolyn A. *Alberto's Exploration of Rational Numbers.* Videotape. New Brunswick, N.J.: Robert B. Davis Institute for Learning, 1994a.

———. *The Development of Fourth-Graders' Ideas about Mathematical Proof.* Videotape. New Brunswick, N.J.: Robert B. Davis Institute for Learning 1994b.

———. *Bits and Pieces: Helping Children Build Ideas about Fractions and Proof.* Videotape, in English or Portuguese. New Brunswick, N.J.: Robert B. Davis Institute for Learning, 1995a.

———. *Investigating Ideas about Fractions.* Interactive CD. New Brunswick, N.J.: Robert B. Davis Institute for Learning, 1995b.

Steencken, Elena. *Explorations to Build Meaning about Fractions: Investigation Units for Preservice Teachers,* Vol. 2., edited by Alice Alston. New Brunswick, N.J.: Robert B. Davis Institute for Learning, 1999.

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Our Rutgers team consisted of Carolyn A. Maher, Amy Martino, the late Robert B. Davis, Elena Steencken, Joan Phillips (a fourth-grade teacher in Conover Road School, Colts Neck, N.J.) and Judith Landis (principal of Conover Road School).

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